

1. When the input to a causal LTI systems is

$$x[n] = -\frac{6}{7} \left(\frac{1}{3}\right)^n u[n]$$

the z-transform of the output is

$$Y(z) = \frac{z}{(1 - \frac{3}{2}z^{-1})(1 - z^{-1})(1 + \frac{1}{2}z^{-1})}$$

- (a) Find the z-transform of  $x[n]$ .
- (b) What is the region of convergence of  $Y(z)$ .
- (c) Find the impulse response of the system.
- (d) Is the system stable? Why or why not?

a

$$\begin{aligned} X(z) &= -\frac{6}{7} \cdot \frac{1}{1 - \frac{1}{3}z^{-1}} + \frac{6}{7} \left( \frac{1}{1 - \frac{2}{3}z^{-1}} \right) \\ &= \frac{-\frac{6}{7}(1 - \frac{2}{3}z^{-1}) + \frac{6}{7}(1 - \frac{1}{3}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 - \frac{2}{3}z^{-1})} = \frac{-\frac{6}{7} + \frac{4}{7} + \frac{18}{14}z^{-1} - \frac{6}{21}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{2}{3}z^{-1})} \\ &= \frac{\frac{54}{42}z^{-1} - \frac{12}{42}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{2}{3}z^{-1})} = \boxed{\frac{z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{2}{3}z^{-1})}} = X(z) \end{aligned}$$

b

The system is causal, so  $Y(z)$  must be causal as its output,  $\therefore Y(z)$  is right-sided

So  $\boxed{\text{ROC: } |z| > \frac{3}{2}}$  ✓



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